TEXTUAL CRITICISM OF DRAWINGS IN HISTORICAL/SCIENTIFIC TREATISE ON REPRESENTATION: THE CAPUT IN DE PROSPECTIVA PINGENDI

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ABSTRACT: The proposed article aims to provide a methodological contribution to reading historical/scientific treatise on representation. In philology, textual criticism is the critical analysis of literary texts aimed at a deep understanding of the text and the author’s intentions. In historical/scientific treatises on representation, the work is composed of two “texts” of equal importance: one is literary, the other graphic. In these cases therefore, textual criticism should be applied to both “texts” at the same time. The case study chosen to investigate the potential of the research method is the first illustrated treatise dedicated to perspective: De Prospectiva Pingendi by Piero della Francesca. While this is a work aimed at painters, the theoretical principles and applications of perspective are also addressed, contributing to a definition of the scientific basis of descriptive geometry. The bibliography is very rich, including essays by C. Winterberg (1899), G. Nicco Fasola (1942), M. Kemp (1994), and K. Andersen (2007). However, it should be noted that graphics that could explain the reasoning behind Piero della Francesca’s drawing are not always included, especially in critical editions. Among the pages of the treatise, proposition 8 in book III holds particular importance for the proposed investigation. Here, Piero constructs the perspective of a human head, which is first positioned straight (frontal position), and then looks upwards (generic position). This proposition represents a major step in that for the first and only time, the author addresses the theme of free forms. To read this proposition critically, it is necessary to refer to a hybrid digital model (polygonal mesh, T-Splines, and NURBS) developed by studying the codices housed in the Palatina Library in Parma and the Municipal Library in Bordeaux. Construction of the digital model allows us to understand both Piero’s refined spatial reasoning and its relative plane rendering in an original reverse path that highlights the scientific nature of the entire treatise and the potential of digital representation as applied to the textual criticism of drawings.

Keywords: Textual criticism of drawings, De Prospectiva Pingendi, Piero della Francesca, perspective, descriptive geometry, history of representation, digital modelling, T-Splines, NURBS, polygonal mesh.

1. INTRODUCTION

Ecdoctics, or textual criticism, a branch of philology, is the criticism of a text, the objective of which is to reconstruct a text in its original form, i.e., the one desired by the author, as well as possible. Textual criticism regards any type of text, from literary to musical texts. The criticism assumes familiarity with other fields of knowledge, which may include the history, literature, language, and philosophy underlying the text. In the case of a literary text, the scope of ecdoctics is to establish the edition of the text itself, possibly through a critical edition. Ecdoctics is formulated in the recognition of “tradition”, that is, the recovery of all material necessary to reconstruct the text. This can be “direct”—when the documents are printed, handwritten, or drafts—or “indirect”—where the documents refer to comments, citations,
imitations, and translations. The direct tradition aims to gather the various testimonies and, by comparing them, establish their chronological order, revealing the various modifications to the text that have occurred over time. When one is in possession of the original, the copies of the text are eliminated in favour of the main testimony. The indirect tradition intervenes to formulate hypotheses and intuitive and interpretational conjectures. A historical scientific treatise is generally composed of two types of texts, a literary one and a graphic one. They each support the other and both hold equal weight in transmitting the author’s thoughts. Making an ecdoctic study of a treatise with these characteristics therefore means putting into practice what was mentioned above, and once the various steps have been performed, a critical edition of the text can be drafted. To do so, it is necessary to consider the two texts because the reading of the literary text can often provide useful information to understand the graphical text; vice versa, the study of the graphical text can often clarify the literary text. It is therefore necessary to observe that while the ecdoctics of an exclusively literary text is a science that has been codified for some time, the ecdoctics of a historical scientific treatise, especially in reference to the graphical text, is taking its first steps with the National Edition (Edizione Nazionale) of the Writings of Piero della Francesca: the *Libellus de quinque corporibus regularibus* [1], the *Trattato d’abaco* [2], and *De prospectiva pingendi*\(^1\). An applied example of the ecdoctics of the drawings in a historical scientific treatise was offered to us by the participation of Riccardo Migliari in drafting the National Edition of *De prospectiva pingendi* by Piero della Francesca\(^2\).

2.2. APPLICATION: *DE PROSPECTIVA PINGENDI*

*De prospectiva pingendi* is a text particularly suited to expressing considerations on the ecdoctics of the drawings. Above all, it is a work known throughout the world and while it is aimed at the exercise of painting, it should be considered a basic testimony of the origins of the science that Gaspard Monge, 300 years later, would call *Géométrie Descriptive*. Indeed, Piero conceives and realizes the first treatise on perspective, documenting it with illustrations like a text on descriptive geometry. The perspective, like the descriptive geometry that forms a part of it, is explained with the drawing and is aimed at constructing an image capable of simulating human vision. The illustration is therefore posed as the result of the science that produced it. In addition, Piero’s work is a historical scientific treatise that is composed of both a literary and a graphical text. In fact, all the propositions contained in the work are explained using these two textual forms [3-5].

Piero della Francesca wrote this, his most famous treatise, between 1470 and 1480, organizing it into three books that list 54 propositions on perspective applied to various problems. The first, composed of 30 propositions, deals with plane figures. The second, composed of 12 propositions, is dedicated to solids. The third, with 12 propositions, regards complex figures [6-8].

There are eight extant copies of the treatise, four of which contain original drawing: Parma and Reggio written in the vernacular and Bordeaux and Milan written in Latin. Of the four

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\(^1\) More specifically, with regard to the ecdoctics of the drawings, a comparison was made of the first two published National Editions of Piero’s treatises [1–2]. The editors of both editions highlighted the dialogue between the text and illustration, remarking how just as the word is capable of evoking an image, the drawing is capable of expressing an idea.

\(^2\) National Edition of *De prospectiva pingendi*, currently being printed by the Istituto Poligrafico e Zecca dello Stato – Rome. Scientific committee: Marisa Dalai Emiliani (President), Ottavio Besomi, Carlo Maccagni; Franca Ela Consolino (Scientific consultant on Latin philology); Riccardo Migliari (Scientific consultant on design). Critical editing of the vernacular text: Chiara Gizzi; Critical editing of the Latin text: Flavia Carderi; Diplomatic and critical editing of the drawings: Riccardo Migliari, Leonardo Baglioni, Federico Fallavollita, Marco Pasolo, Matteo Flavio Mancini, Jessica Romor, Marta Salvatore; Edoctic contributions: Alessandra Sorci.
remaining copies, Milan and Bologna are in the vernacular and London and Paris are in Latin\(^3\).

As mentioned above, our participation in drafting the volume of drawings for the National Edition of *De prospectiva pingendi* allowed us to tackle the problems posed by the ecdoctics of the drawings, comparing three cases that are briefly described below. In the first case, the drawing simply reinforces what is expressed in the text, helping to transmit the idea. In the second case, the drawing integrates the text where the text relies on the illustration to complete the understanding of the proposition. In the third case, the drawing cannot completely transcribe the text due to the complexity of the construction and the reduced size of the codex.

![Figure 1: Comparison between the Bordeaux codex, the diplomatic edition and the critical edition.](image)

Respecting what was already done in the National Editions of the *Libellus* and the *Abaco*, we created two different transcriptions for each drawing in the treatise: a diplomatic and a critical draft. The former aimed to reproduce the original drawing as well as possible. It was therefore recreated in the original dimensions, marks were decomposed into lines of variable width to simulate the speed of the trace, highlighting the starting and stopping points, and dry incisions were also reproduced. The critical draft was aimed at transmitting the message of association of the text and the drawing. In this case as well, the drawing was made in the original dimensions and an enlargement was made when necessary in order to facilitate the understanding of some of the more complex steps.

The critical edition of the drawings is undoubtedly the most stimulating of the two drafts in that it tends to translate the meticulous indications of the text into signs, inviting the reader to follow the construction step by step. This ambitious objective consists in clearly and completely expressing the intellectual characteristics of the work through the practice of drawing.

This type of study also allowed us to further highlight the system used by the author to build his perspectives. Piero makes use of two

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\(^3\) Ms. Parmense 1576, Biblioteca Palatina, Parma; Ms. Reggiano A 41/2 (previously A 44), Biblioteca Comunale “Antonio Panizzi”, Reggio Emilia; Ms. D 200 inf., Biblioteca Ambrosiana, Milan; Ms. A266, Biblioteca dell’Archiginnasio, Bologna; Ms. 616, Bibliothèque Municipale, Bordeaux; Ms. S.P. 6 bis (previously C. 307 inf.), Biblioteca Ambrosiana, Milan; Ms. Additional 10366, The British Library, London; Ms. Lat. 9337, Bibliothèque Nationale de France, Paris.
“modes”. The first, which we call “method”, refers to the perspective built with the direct method and therefore with operations made directly on the picture plane; this method is applied to the first and second book. The second mode, which we indicate with the term “procedure” in reference to the “architects’ procedure”, is based on building the perspective using other representations such as the plan and the elevation; the propositions presented in the third book are developed through this procedure.

Of the eight copies of the treatise, our attention is directed mainly at those in Bordeaux and Parma because they both contain original drawings and because they are representative of the dual Latin/vernacular tradition. But this choice did not preclude the possibility, or even sometimes the need, to also consult the other codices.

As an example of what is expressed above, this contribution presents a proposition from the third book that is particularly complex and meaningful (Fig. 1).

The choice of proposition 8 from the third book as a case study depends on the particular characteristics represented by the theme caput. In the first place, the natural representation of the head is one of the most difficult and frequent challenges that a painter must face, particularly in the realization of monumental works, for example, the fresco cycle of *The Legend of the True Cross* that the same Piero della Francesca painted in the basilica of San Francesco in Arezzo around the middle of the fifteenth century. The correct representation of the large variety of positions that the human head can take is evidently a complex problem and this could explain the reason why Piero della Francesca decided to deal both with the perspective of an upright head and with the head in a general position in this proposition of *De prospectiva pingendi*.

In the second place, the human head presents an original geometrical characteristic with respect to all the other subjects addressed in the treatise. It is the first and only free form, characterized by continuous curvature that excludes the presence of edges and vertices.

This geometric characteristic motivates the effort to discretize the form that Piero describes minutely and which leads to the identification of the univocal position of 162 points on the surface of the head. By representing the positions assumed by these points during the different operations, Piero can control the movements of the head and trace the perspective correctly.

Given the characteristics of the proposition and the objectives of the critical analysis—identify and remove possible errors and contradictory elements, retracing and understanding in depth the thoughts and intentions of the author—a 3D model of the head was reconstructed. The model allowed a perfectly coherent 2D graphical apparatus to be obtained and to retrace step by step the operations described in the literary text, which were sometimes only partially represented by the drawings present in the codices. The possibility of repeating these operations in a digital model also suggested the hypothesis that Piero della Francesca himself made use of a model to experiment three-dimensionally with the different actions necessary to finish the perspective construction.

In this paper we focus on the methodological system proposed rather than on the careful description of the individual steps of the proposition. We therefore now address two important reflections regarding the 3D digital model and its relationship with the drawings presented in the codices.

First, it is important to enquire what characteristics the 3D digital model should have in order to correctly represent the subject of this proposition and to satisfy the need for a critical analysis. Above all, the model should be capable of correctly representing an organic, completely free surface like the human head. It should then allow exact geometric operations such as measurements, sections, and

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4 For a description of the steps that follow, see the Bordeaux codex.
projections to be made. The first of these properties is typically assigned to polygonal modelling, which uses mesh surfaces and allows for a sculptural approach to the modelling. In contrast, the execution of exact geometric operations is precisely NURBS modelling, in which the continuous surface, being represented through polynomial equations, can describe the geometrical properties of each point on the surface.

The presence of both characteristics can be traced to T-Spline geometry, a family of NURBS geometries that integrates some properties of the mesh surfaces and allows a sculptural approach to be taken when modelling parametric NURBS surfaces [9-10] (Fig. 2).

Figure 2: Comparison between the caput 3D model by means of polygonal meshes, T-Spline surfaces and NURBS surfaces.

Second, it is necessary to evaluate the relationship this model should have with Piero’s original drawings. This means understanding if these drawings are appropriate or not to use as the basis for creating a three-dimensional model. The usual strategy when modelling a freeform surface is to consider the elevations and a series of parallel sections to build a three-dimensional reference grid, an operation that is made possible due to the profile drawings and the horizontal sections of the head called circuli. The rendering in space by redrawing these traces returns a grid alluding to the shape of a head but which shows elements of notable incoherence between the horizontal sections and elevations (Fig. 3).

The lack of correspondence between elevations and horizontal section renders these drawings inappropriate for use in their current state for the construction of the model. With the aim of conserving the original data provided by the original drawings, the preceding redrawn traces were modified to make them coherent, respecting the original trace given by Piero della Francesca as much as possible. This second set of traces was used as the basis to construct a 3D model. This model, showed evident anomalies, with respect to both a correct anatomic model and the perspective image represented in the codex: the nasal septum is excessively wide and the forehead squared, while the cheeks seem emaciated (Fig. 4).

Figure 3: The horizontal sections don’t intersect the elevation of the caput.

Figure 4: The first model of the caput shows an incorrect anatomic appearance.
The difficulties found in these first experiments highlighted the allusive character that the drawings present in the codices of *De prospectiva pingendi* sometimes take and they confirmed the need to always read the literary and the graphical texts simultaneously. We therefore decided to set aside the eight horizontal sections traced by the author and to use only the elevations—front and side—to create the 3D model of the head. In addition, all modelling followed the scheme of a credible anatomic model. The new horizontal sections were then extracted from this model, which makes them perfectly coherent with the elevations and with all the two-dimensional studies that describe the different phases of the proposition.

To trace the perspective of the *caput*, Piero della Francesca decided to use the second of the two modes shown in the *De prospectiva pingendi*, the one he considered more appropriate for the representation of complex forms.

The procedure can be divided into three steps: discretization of the form, in which the univocal positions of the 162 points pertaining to the surface of the head are identified (Fig. 5); projection and sectioning of the points on the picture plane, which allows the perspective images of the points to be identified thanks to their horizontal coordinates, called *latitudes*, and vertical coordinates, called *altitudes* (Fig. 6); and drawing the perspective, which occurs by intersecting the horizontal and vertical coordinates (Fig. 7).

The perspective image of the *caput* presents apparent deformations in the left part of the forehead that would no longer be visible observing the perspective from its centre of projection. Piero della Francesca’s second mode, however, does not provide information to reconstruct the spatial position of the centre of projection starting just from the perspective image, even if these data can be obtained from the drawing that describe the preceding phases of the construction.
The second part of the proposition continues with a series of rigid transformations applied to the upright head, which situate it in a generic position. For this reason the head in its generic position can be considered the second part of proposition 8 and not a proposition in and of itself. Piero suggests taking the profile of the upright head and applying a first rotation to it so that it is inclined backwards, so it looks upwards. The superposition of the two profiles instead shows that these are two different drawings. This aspect provides useful indications as to Piero’s modus operandi, which confirms that the author made use of reference models to draft the drawings present in the pages of the treatise. It is now convenient to pause on the nature of the spatial rotations that are implemented. On the one hand, they demonstrate surprising control of the method of plan and elevation representation; on the other, they confirm once again how the use of digital representation makes it possible to simultaneously read the text carefully and deeply. For convenience we use a reference system defined by three orthogonal planes: $\pi_1$, $\pi_2$, and $\pi_3$ (Fig. 8). As mentioned above, the first rotation was applied to the upright head so that each of its notable points traces out circular arcs parallel to the $\pi_3$ plane. There is a detail in this first movement that could be overlooked if the drawings were not made according to the steps in the written text. In fact, the last four circuli, E, F, G, and H fold and go from straight to skew during the rotation. It almost seems as if the cylindrical surfaces they pertain to describe the natural movement of the cervical vertebrae of a human head connecting the axis of the spinal column with the axis of the head. This aspect is not described in the literary text, but is evident when analysing the drawings in the four original codices of the treatise (Fig. 9).

5 “Habes caput oculi unius quod constat per id quod precessit”.

6 This aspect is further confirmed in the presence of numerous holes in the pages of the manuscript, which were probably used to transfer the reference models.

Figure 8: Spatial rotations applied to the upright head which situate it in a generic position.
Figure 9: Comparison between the four original codices about the last *circuli*.

We now return to the construction. To be able to draw the front elevation of the inclined head, Piero suggests considering the distance between each individual point of each section and the head’s plane of symmetry. These distances can be measured on the front elevation of the upright head because they do not vary following the first rotation. The reasoning proposed, which is impeccable from the geometrical point of view, was already addressed in proposition 5 of the third book, in which a cube, following a sequence of spatial rotations, is represented in a generic position resting on one of its vertices.

The logic of the operations is the same as the one proposed for the proposition of the inclined *caput*, but in the case of the cube as well, the demonstration of the correctness of the operations is entrusted entirely to the drawing and its heuristic and experimental potential. It is noteworthy that the rotations made by Piero sometimes alternate with the description of constructive phases useful to determining the orthogonal projections of the oriented lines and points following such transformations. The second rotation occurs making the head lean towards one side. This is an anticlockwise rotation in which the points trace out circular arcs parallel to the $\pi_2$ plane and it is precisely in this phase that Piero proposes a refined way to resolve the drawing of the first projection of the *circuli*, which are shortened following the first and second rotations. The author proceeds in the following way: he situates the profile drawing of the head looking upwards (rotation 1) orthogonal to the elevation of the inclined, reclined head (rotation 2). To intersect the traces from this arrangement of the drawings, Piero draws the first projection of the points and the *circuli*. But what spatial configuration corresponds to this organization in the drawings? In this phase as well, the digital drawing is made explicit directly in the virtual space, providing its contribution and making us appreciate the maturity of Piero’s reasoning. The profile drawing does not represent a side view of the inclined head, but rather an orthogonal projection onto a plane parallel to the head’s plane of symmetry, which is then turned over onto the ichnographic (horizontal) plane (Fig. 10).

The front elevation of the inclined, reclined head is a drawing that pertains to the orthographic (vertical) plane, which is then turned over onto the ichnographic plane. As a consequence, the procedure is exemplary from the geometrical point of view. The last rotation of the head, which leads to the most generic position possible, is addressed directly in the drawing instrumental in calculating the *latitudes*, and therefore in the phase of the perspective projection in the first projection. The plans of the *circuli* just calculated (divided into four groups to make the drawing more legible) are situated obliquely to the picture plane. In this rotation, the points trace out circular arcs parallel to the horizontal $\pi_1$ plane and precisely because their height does not vary, they can be measured in the front elevation of the inclined, reclined head and used to calculate the intersection of the visual pyramids in the *latitudes*. The construction ends with the drawing of the perspective of the head in a generic position obtained by interpolating the perspectives of
the notable points, obtained as always by intersecting the horizontal and vertical coordinates of the notable points (Fig. 11).

3. CONCLUSIONS AND POSSIBLE DEVELOPMENTS

The case study from De Prospectiva Pingendi briefly presented here highlights the analytical and experimental potential of the eudocitcs of the drawings in this historical scientific treatise of representation, defining some of its main characteristics, and especially its “clarifying character”, which allows for the complete, effective, interpretational capacity of the treatise under study. We point, for example, to the drawings depicting the altitudes of the inclined head (Fig. 12). To an initial, superficial reading, the curved lines shown would seem to describe the arrangement of the circuli of the head subject to the three rotations. But only by means of a redrawing proceeding in step with the reading of the text and realized with the principles and tools listed above can their effective nature of being simple operational signs be seen, with the scope of separating the groups of notable points of each circulo constitutis terminis recedatur, veluti in alterius capitis circumscriptione positum prefuit”.

Figure 10: Spatial configuration to construct the first projection of circuli after two rotations.

Figure 11: The perspective of the caput in generic position.
simply to make them more legible. In fact, upon further thought, the drawing of the elevation of the head shortened according to the three rotations, would present great difficulty in execution.

Figure 12: Clarifying character of critical edition (on the right) compared to the diplomatic drawing (on the left).

Piero therefore decides that in that phase, the drawing should only provide the operational data of the position of the notable points in space necessary to correctly follow the construction.

The “educational character” of the method proposed thus emerges, which allows particularly effective and communicational critical drawings to be realized, which are even accessible to scholars in different disciplines. Always regarding the same aspect, one could think of a purely digital critical edition of a treatise aimed at teaching its nature, which could allow the scholar to control and manipulate the 3D models representing the construction phases (Fig. 13). In this way, the critical drawing highlights the spatial properties of the author’s reasoning, thus going on to define the “demonstrative character” of the ecdoctics applied to the drawings. In this regard, we recall the cited spatial reasoning proposed by Piero in the rotation of the head in which the experimental demonstration is completely transferred to the strength of the lines. Finally, the “experimental character” of a method should not be forgotten.

Figure 13: The caput by Piero della Francesca and its critical edition.
Based on digital technology, this allows new representations to be conceived and realized, such as 3D printing, augmented reality applications, which, with original contributions, can enrich the thoughts of the author and his work.

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REFERENCES


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